# Lecture 5: Graphs.

Graphs!

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Euler

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Graphs!
- Euler
- Definitions: model.

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- Euler
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- Fact!

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- Euler Again!!

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Planar graphs.

# Lecture 5: Graphs.

Graphs!
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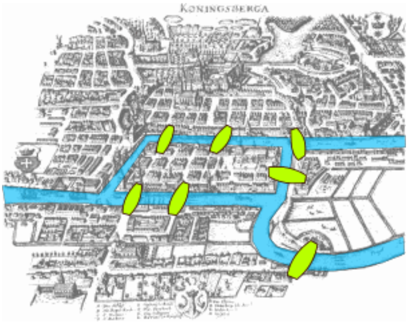
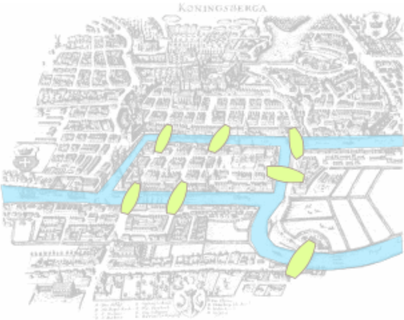
Planar graphs.
  Euler Again!!!!

# Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

“Konigsberg bridges” by Bogdan Giuşcă - License.



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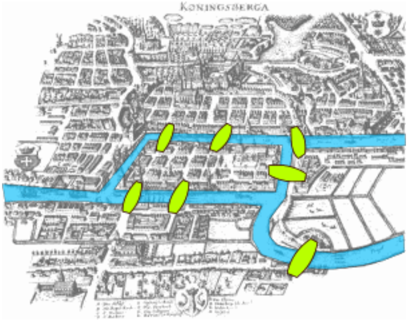


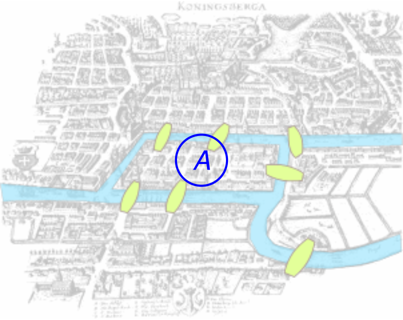

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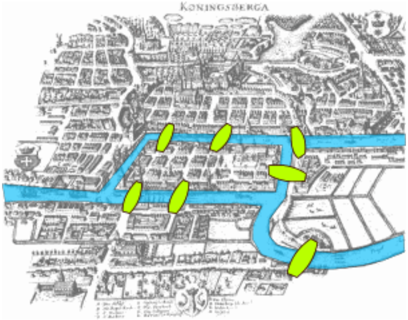


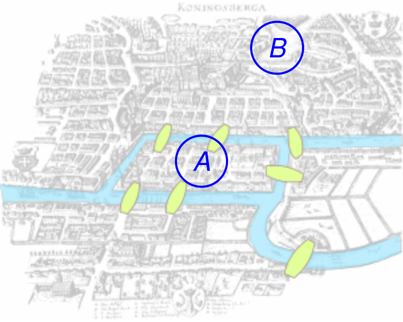

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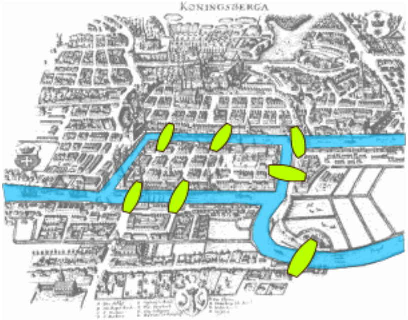


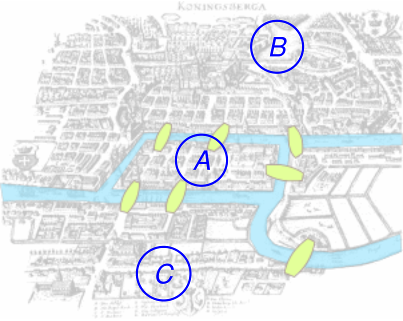

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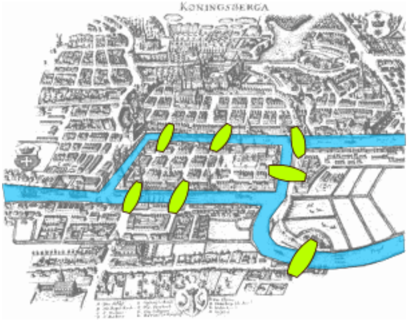


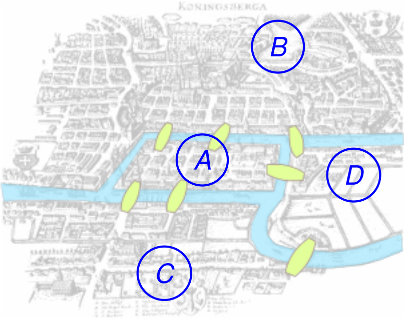

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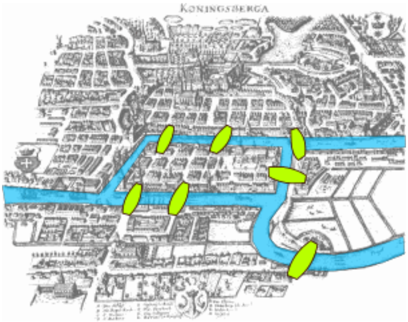

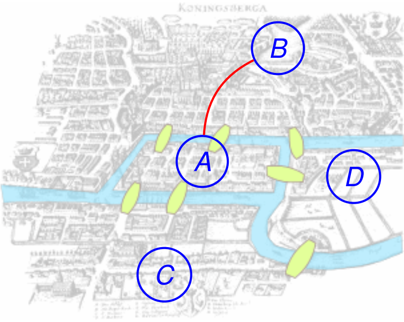

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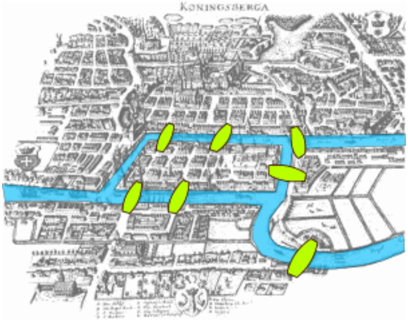


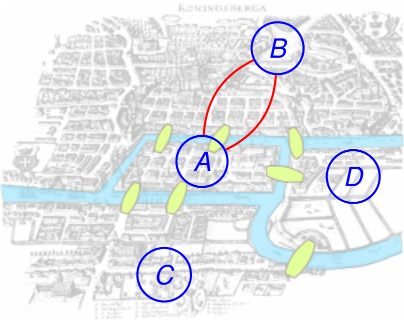

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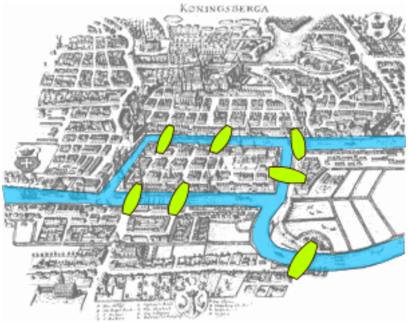


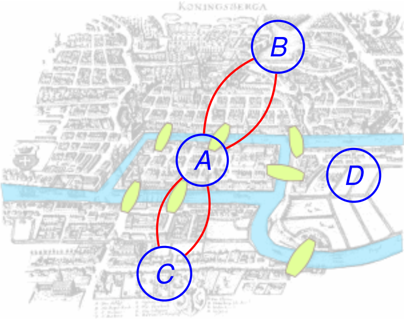

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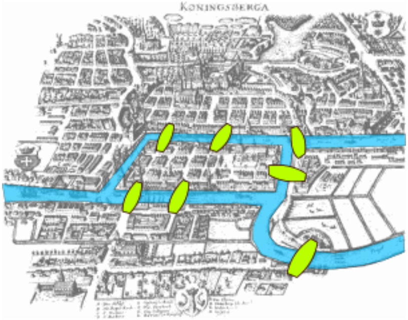


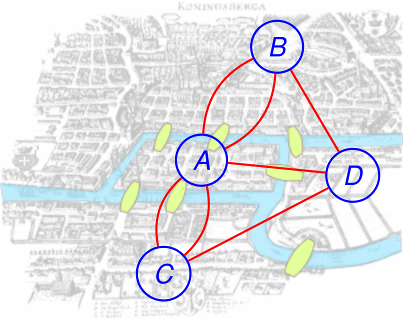

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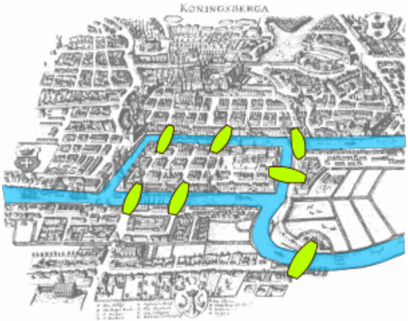


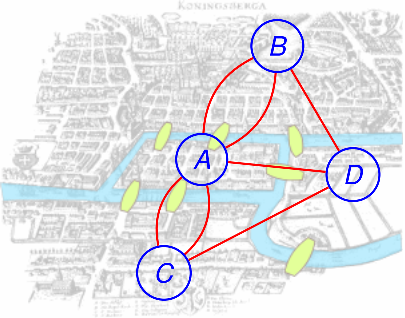


Can you draw a tour in the graph where you visit each edge once?

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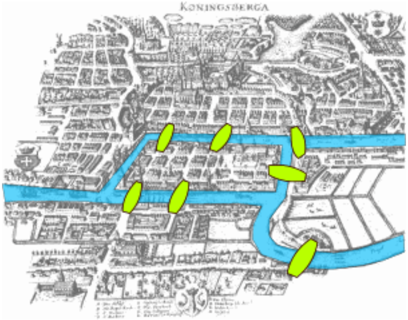

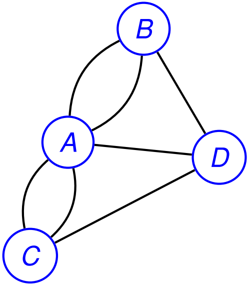


Can you draw a tour in the graph where you visit each edge once?
Yes?

# Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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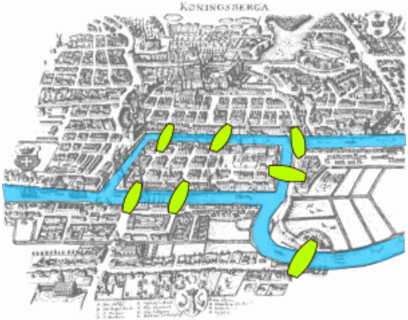


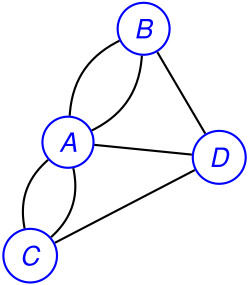


Can you draw a tour in the graph where you visit each edge once?
Yes?  No?

# Konigsberg bridges problem.

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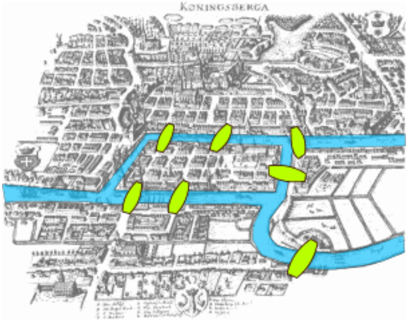
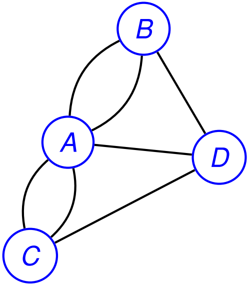
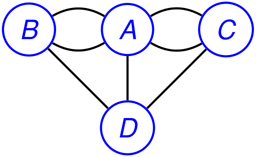


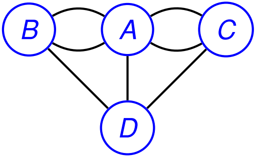

Can you draw a tour in the graph where you visit each edge once?
Yes? No?
We will see!

# Graphs: formally.



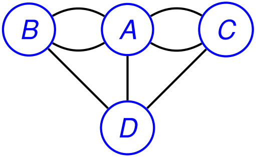

Graph:

# Graphs: formally.




Graph: $G = (V, E)$.

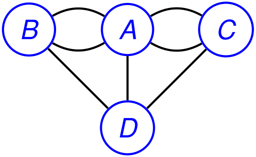
# Graphs: formally.




Graph: $G = (V, E)$.

$V$ - set of vertices.

# Graphs: formally.

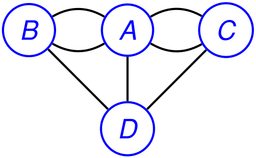



Graph: $G = (V, E)$.

$V$ - set of vertices.

$\{A, B, C, D\}$

# Graphs: formally.




Graph: $G = (V, E)$.

$V$ - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ -

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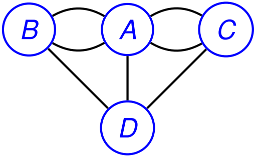


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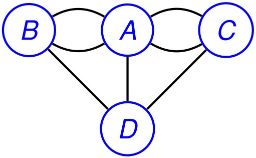


Graph: $G = (V, E)$.

$V$ - set of vertices.

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$\{\{A, B\}$

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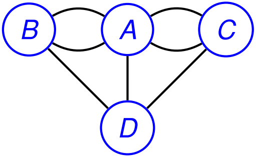


Graph: $G = (V, E)$.

$V$ - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}$

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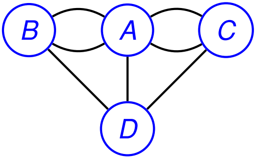


Graph: $G = (V, E)$.

$V$ - set of vertices.

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$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}, \{A, C\},$

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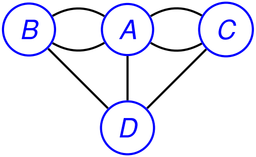


Graph: $G = (V, E)$.

$V$ - set of vertices.

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$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}, \{A, C\}, \{B, C\}, \{B, D\}, \{B, D\}, \{C, D\}\}$.

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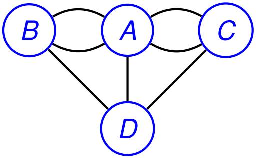


Graph: $G = (V, E)$.

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$\{\{A, B\}, \{A, B\}, \{A, C\}, \{B, C\}, \{B, D\}, \{B, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

# Graphs: formally.


Graph: $G = (V, E)$.

$V$ - set of vertices.

$\{A, B, C, D\}$

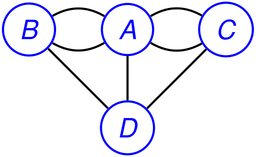
$E \subseteq V \times V$ - set of edges.

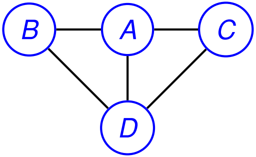
$\{\{A,B\},\{A,B\},\{A,C\},\{B,C\},\{B,D\},\{B,D\},\{C,D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

# Graphs: formally.




Graph: $G = (V, E)$.

$V$ - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

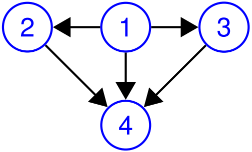
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{B, C\}, \{B, D\}, \{B, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

Multigraph above.

# Directed Graphs




$G = (V, E)$.

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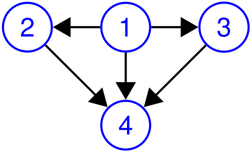


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$V$ - set of vertices.

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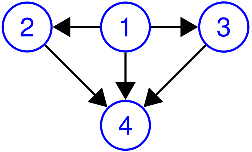
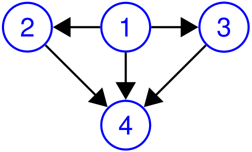



$G = (V, E)$.

$V$ - set of vertices.

$\{1, 2, 3, 4\}$

# Directed Graphs



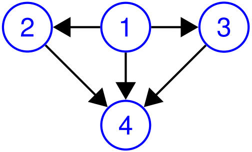

$G = (V, E)$.

$V$ - set of vertices.

$\{1, 2, 3, 4\}$

$E$ ordered pairs of vertices.

# Directed Graphs

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$\{(1, 2),$

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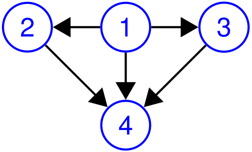


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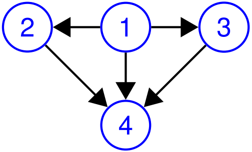


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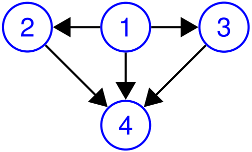


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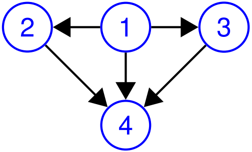


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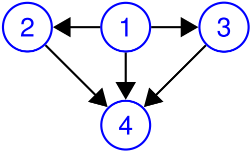
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.

# Directed Graphs




$G = (V, E)$.
$V$ - set of vertices.
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$E$ ordered pairs of vertices.
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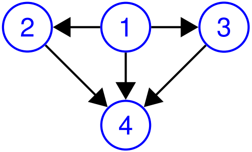
One way streets.
Tournament:

# Directed Graphs

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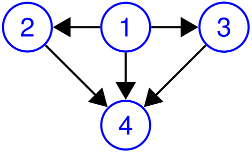
One way streets.

Tournament: 1 beats 2,

# Directed Graphs

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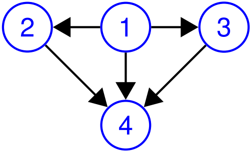
One way streets.
Tournament: 1 beats 2, ...
Precedence:

# Directed Graphs

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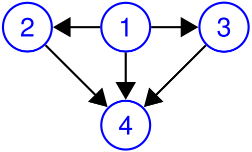
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2,

# Directed Graphs

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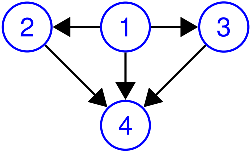
$\{1, 2, 3, 4\}$

$E$ ordered pairs of vertices.

$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

# Directed Graphs

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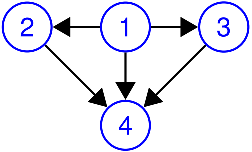
# Directed Graphs

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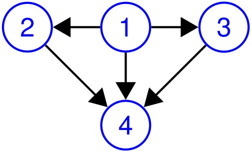


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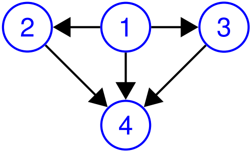


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Friends.

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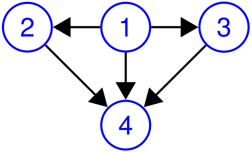


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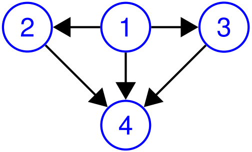
# Directed Graphs

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Social Network: Directed? Undirected?
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Likes.

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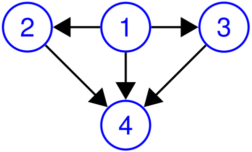


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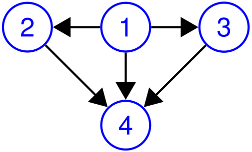


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# Graph Concepts and Definitions.

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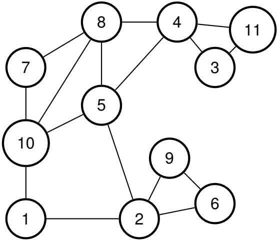


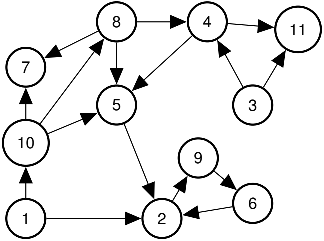


Neighbors of 10?

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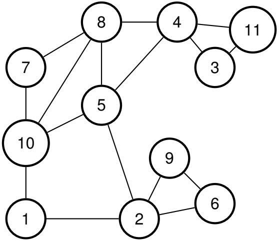


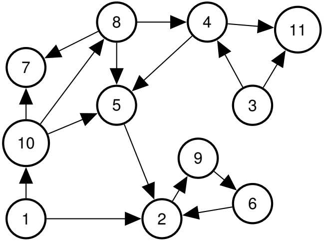


Neighbors of 10? 1,

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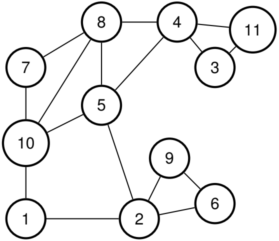


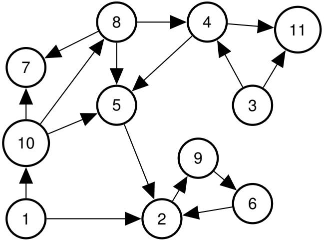


Neighbors of 10? 1,5,

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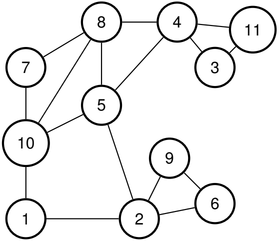


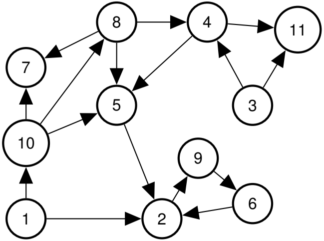


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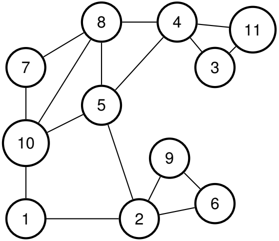


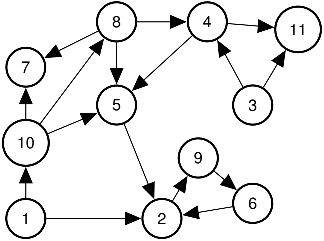


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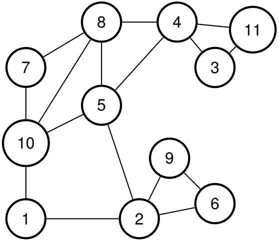


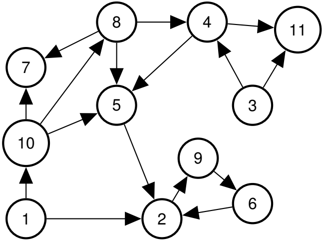


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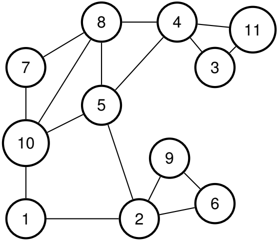


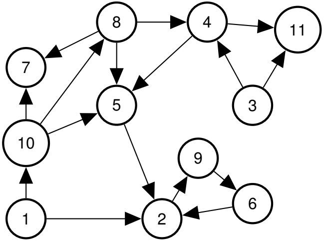


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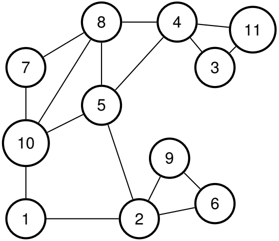


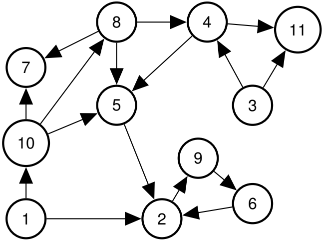


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Degree of vertex 1?

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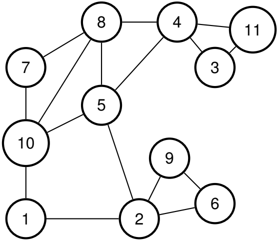


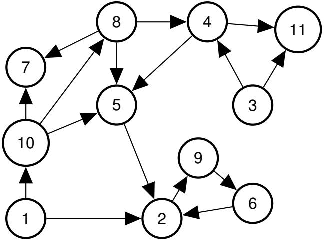


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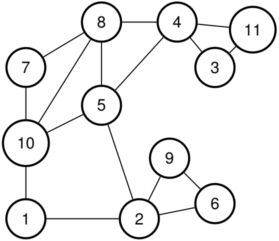


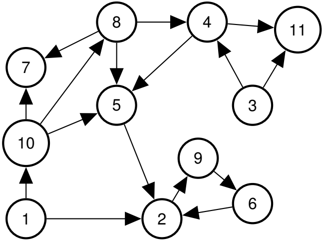


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Degree of vertex $u$ is number of incident edges.

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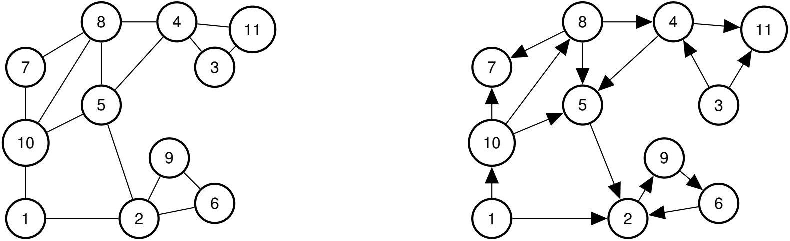


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Equals number of neighbors in simple graph.

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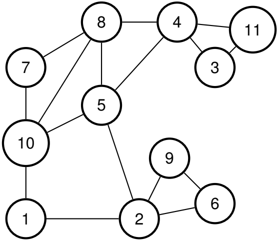


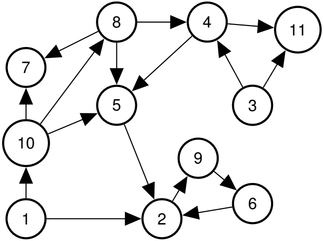


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**Directed graph:**

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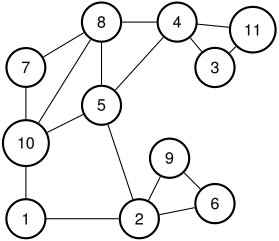


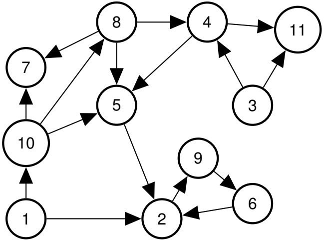


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Degree of vertex 1? 2

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**Directed graph:** In-degree of 10?

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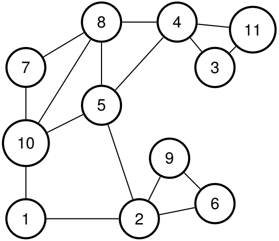


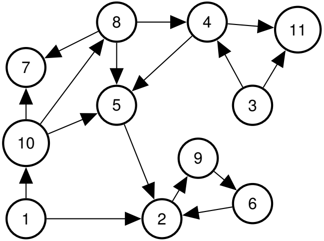


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**Directed graph:** In-degree of 10? 1

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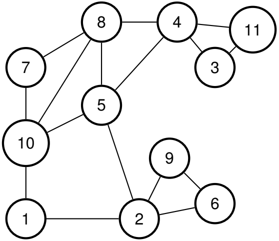


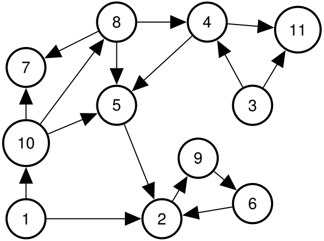


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**Directed graph:** In-degree of 10? 1    Out-degree of 10?

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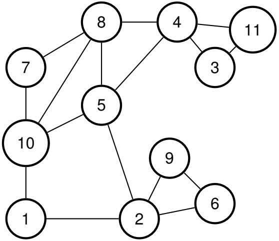


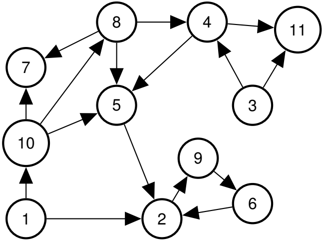


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**Directed graph:** In-degree of 10? 1    Out-degree of 10? 3

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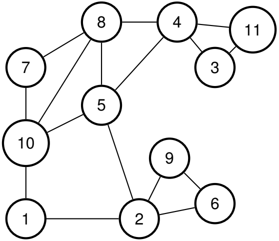


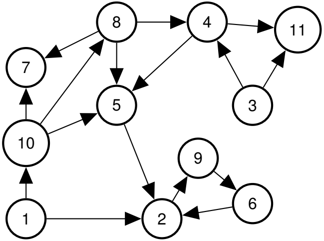


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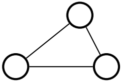

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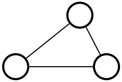

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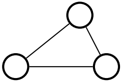

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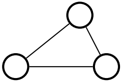


Not (B)! Triangle.

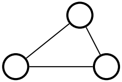
What?

# Quick Proof.

The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.
(B) the total number of edges, $|E|$.
(C) What?

Not (A)! Triangle.



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What? For triangle number of edges is 3, the sum of degrees is 6.

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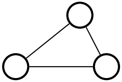

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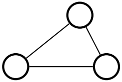

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Could it always be...$2|E|$? ..

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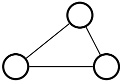

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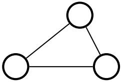

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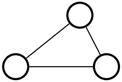

Not (B)! Triangle.

What? For triangle number of edges is 3, the sum of degrees is 6.

Could it always be...$2|E|$? ..or $2|V|$?

How many incidences does each edge contribute? 2.

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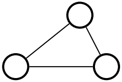

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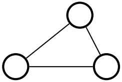

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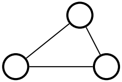

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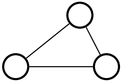

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sum of degrees is total incidences

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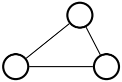

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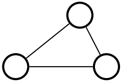

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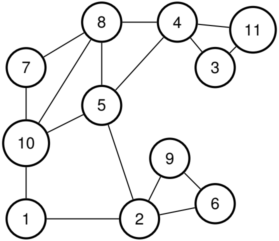
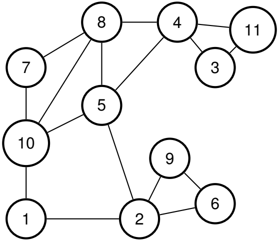
**Thm:** Sum of vertex degress is $2|E|$.

# Paths, walks, cycles, tour.




A path in a graph is a sequence of edges.

# Paths, walks, cycles, tour.

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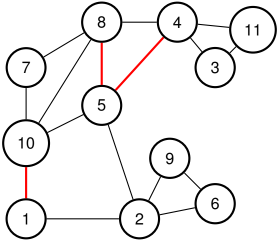


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Path? $\{1, 10\}$, $\{8, 5\}$, $\{4, 5\}$ ?

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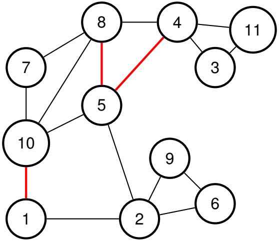


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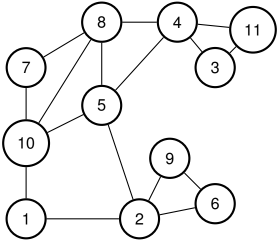


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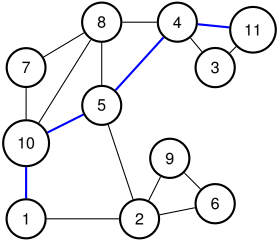


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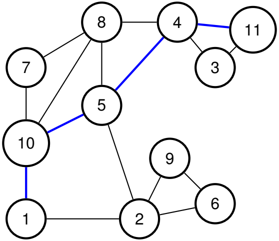


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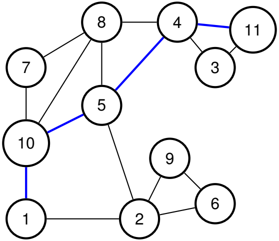


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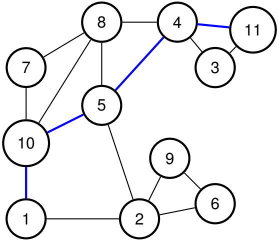


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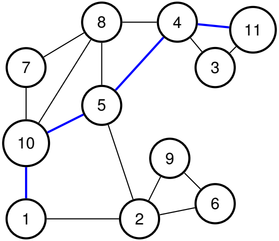
Quick Check!

# Paths, walks, cycles, tour.

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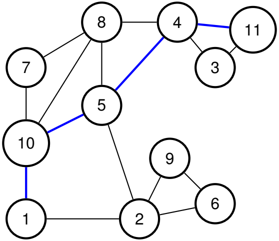
Quick Check! Length of path?

# Paths, walks, cycles, tour.

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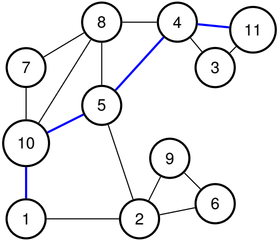
Quick Check! Length of path? $k$ vertices

# Paths, walks, cycles, tour.

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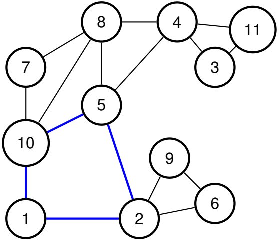
Quick Check! Length of path? $k$ vertices or $k-1$ edges.

# Paths, walks, cycles, tour.

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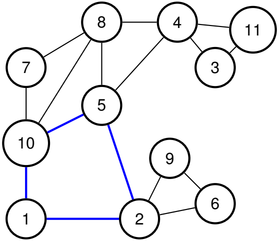
Quick Check! Length of path? $k$ vertices or $k-1$ edges.

Cycle: Path with $v_1 = v_k$.

# Paths, walks, cycles, tour.

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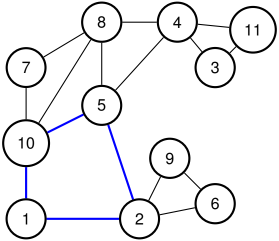


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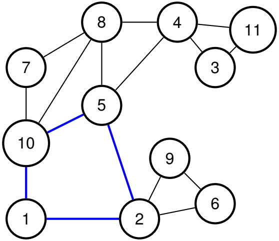
# Paths, walks, cycles, tour.

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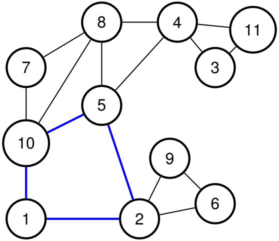
Cycle: Path with $v_1 = v_k$. Length of cycle? $k-1$ vertices and edges!
Path is usually simple.

# Paths, walks, cycles, tour.

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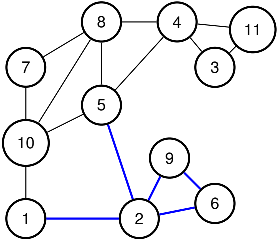


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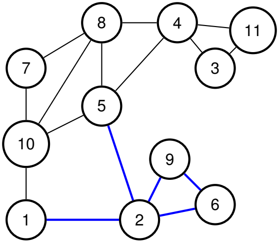
Walk is sequence of edges with possible repeated vertex or edge.

# Paths, walks, cycles, tour.

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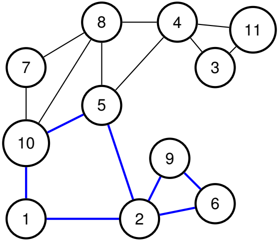


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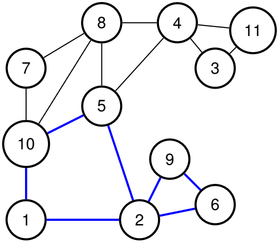
Tour is walk that starts and ends at the same node.

# Paths, walks, cycles, tour.

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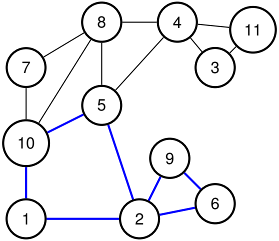


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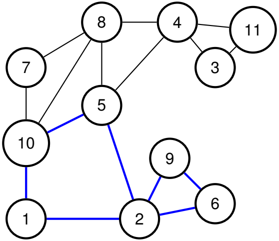
Quick Check!

# Paths, walks, cycles, tour.

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Quick Check! Length of path? $k$ vertices or $k-1$ edges.

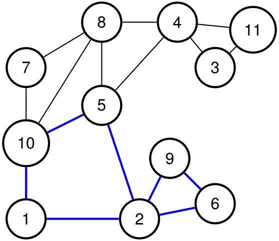
Cycle: Path with $v_1 = v_k$. Length of cycle? $k-1$ vertices and edges!
Path is usually simple. No repeated vertex!

Walk is sequence of edges with possible repeated vertex or edge.

Tour is walk that starts and ends at the same node.

Quick Check!
Path is to Walk as Cycle is to ??

# Paths, walks, cycles, tour.

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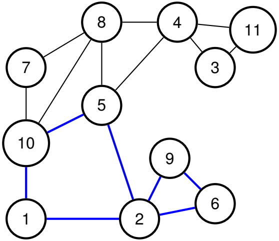


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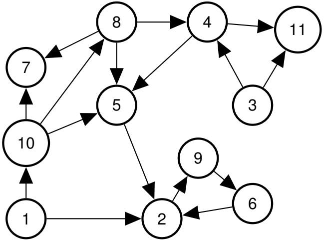

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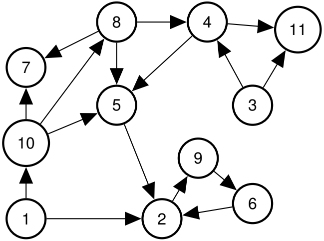


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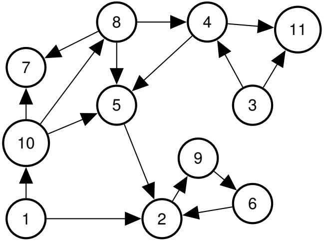


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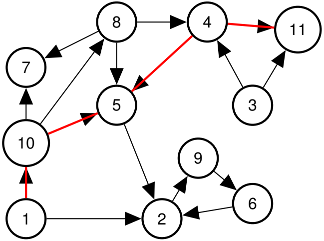


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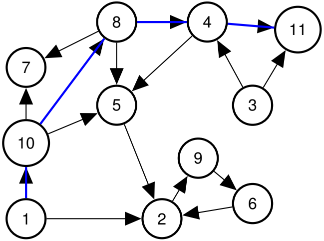


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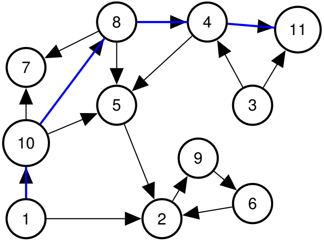


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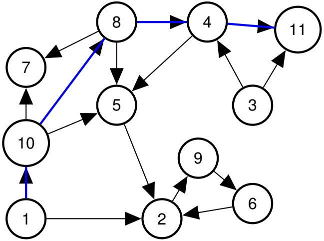


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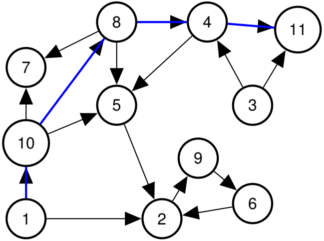


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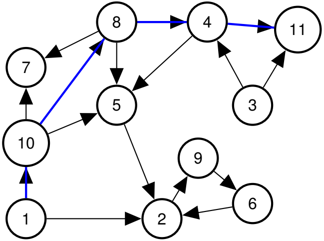


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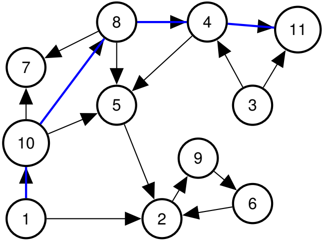


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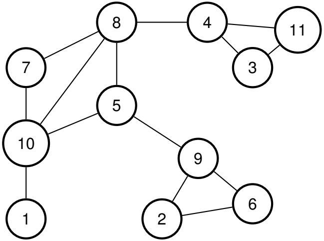
# Connectivity

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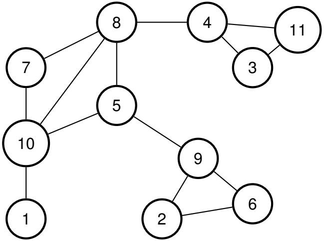


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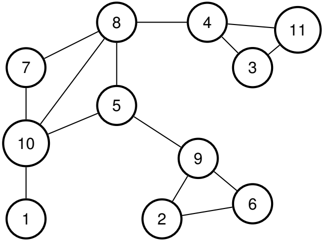


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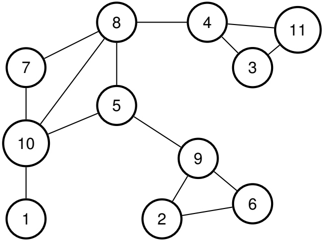


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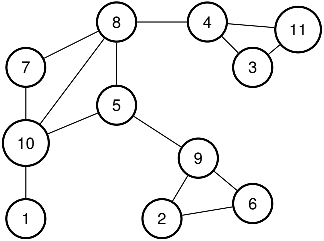


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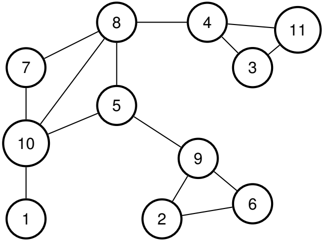


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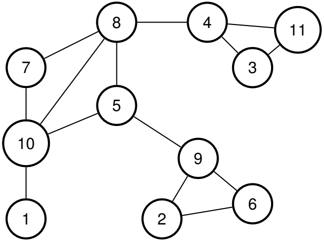


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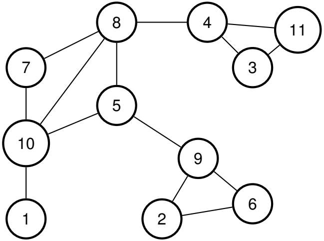


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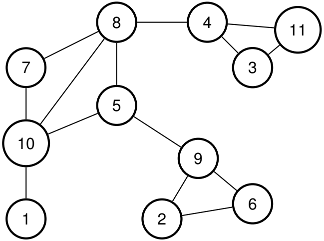


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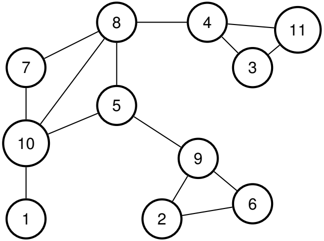


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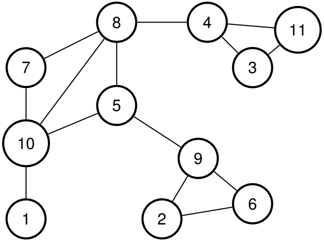


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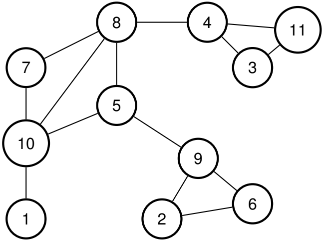


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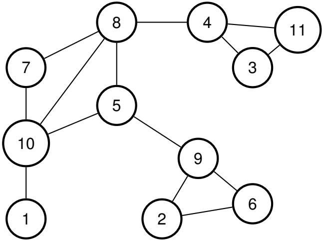


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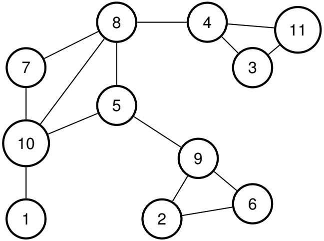


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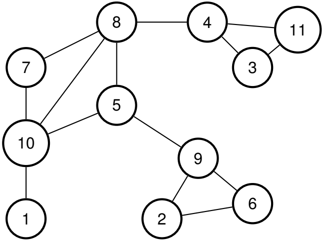


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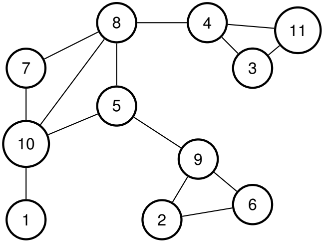


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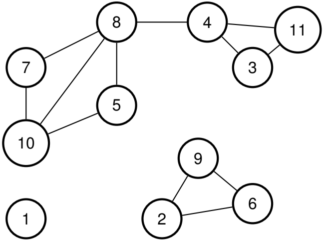


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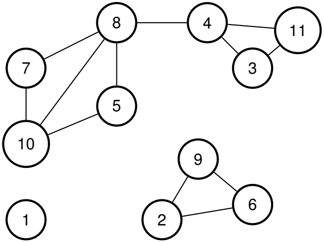


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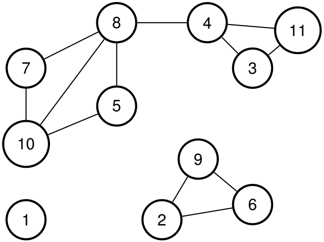


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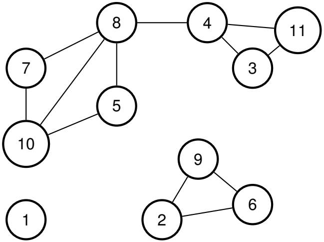


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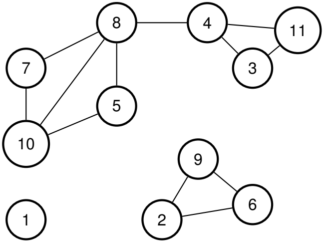


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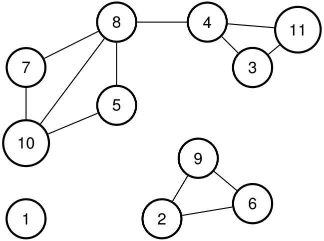


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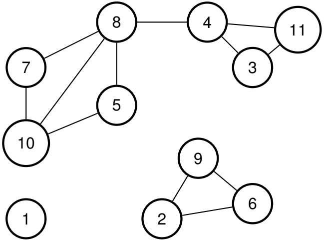


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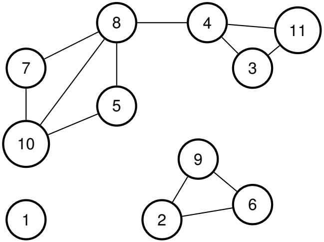


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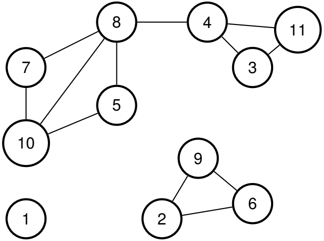


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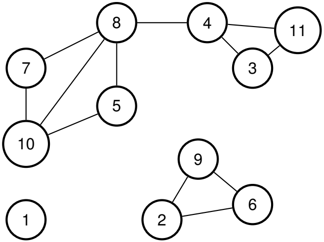


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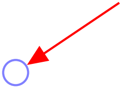

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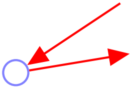

When you enter, you leave.

# Finally..back to Euler!

An Eulerian Tour is a tour that visits each edge exactly once.

**Theorem:** Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

**Proof of only if: Eulerian $\implies$ connected and all even degree.**
Eulerian Tour is connected so graph is connected.
Tour enters and leaves vertex $v$ on each visit.
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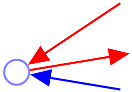

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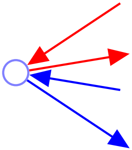

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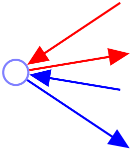

When you enter, you leave.
For starting node,

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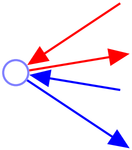

When you enter, you leave.
For starting node, tour leaves first

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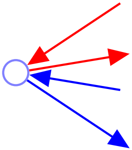

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For starting node, tour leaves first ....then enters at end.

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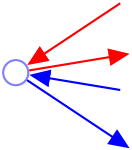

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**Proof of if: Even + connected $\implies$ Eulerian Tour.**
We will give an algorithm.

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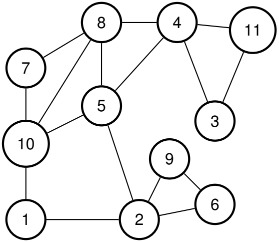

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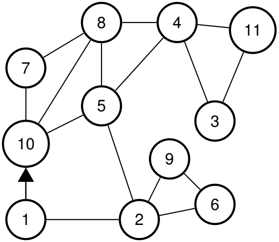


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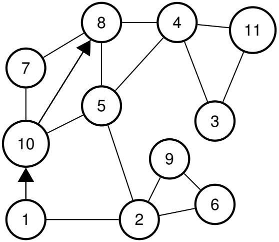


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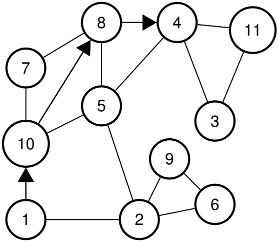


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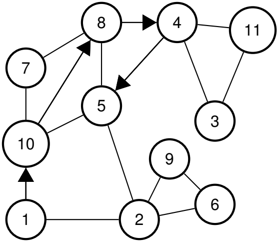


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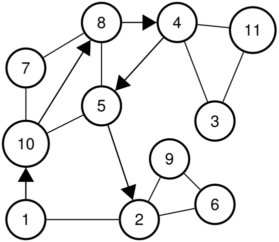


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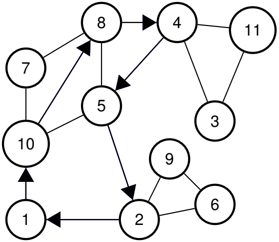


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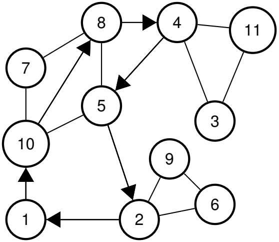


1. Take a walk starting from $v$ (1) on “unused” edges
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2. Remove tour, $C$.

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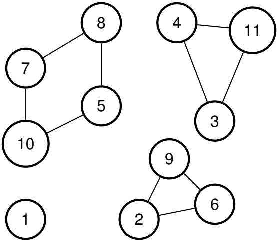


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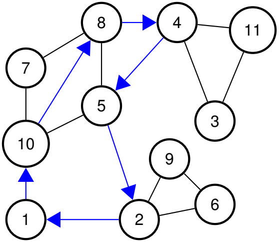


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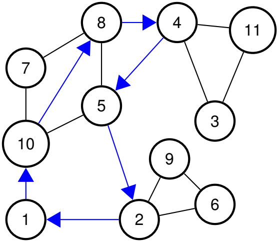


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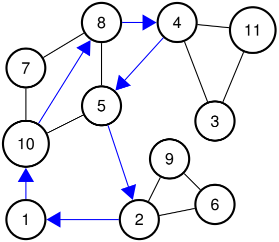


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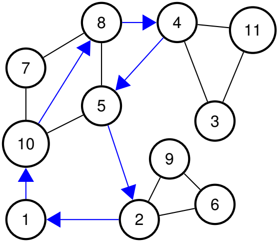


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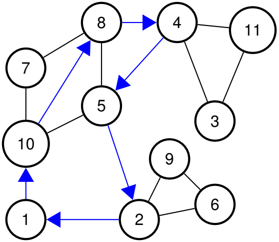


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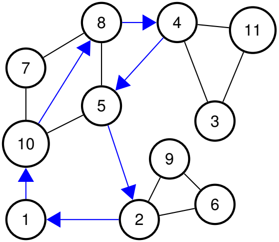


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   Example: $v_1 = 1$, $v_2 = 10$,

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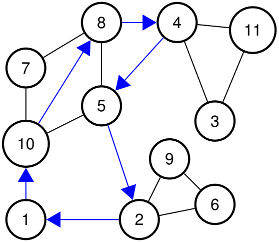


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   Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$,

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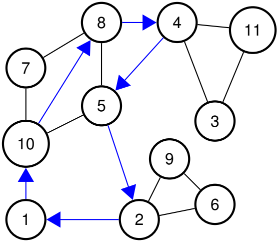


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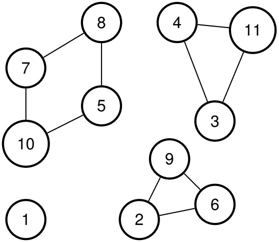


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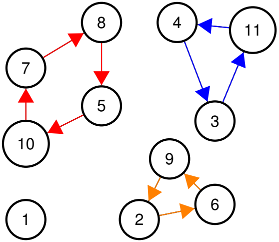


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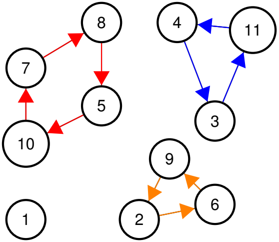


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5. Splice together.

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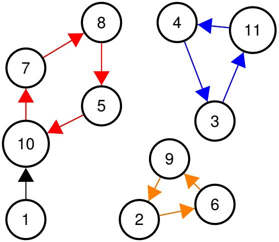


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   1,10

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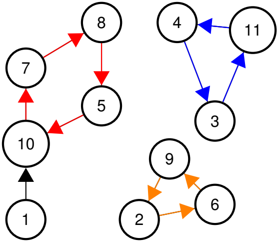


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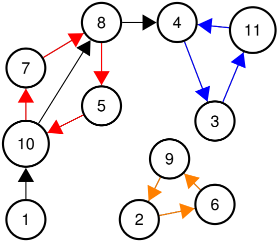


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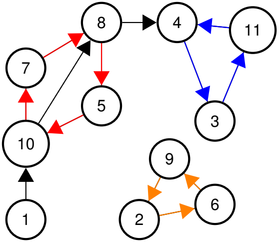


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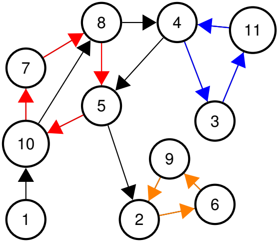


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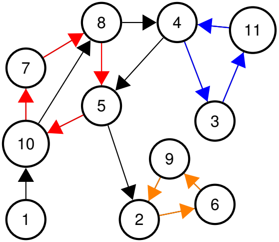


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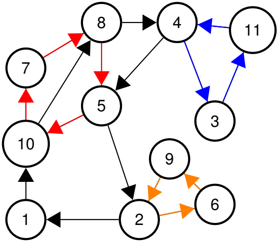
**Proof of if: Even + connected $\implies$ Eulerian Tour.**
We will give an algorithm. First by picture.




1. Take a walk starting from $v$ (1) on "unused" edges
... till you get back to $v$.
2. Remove tour, $C$.
3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by $C$.
      Why? $G$ was connected.
   Let $v_i$ be (first) node in $G_i$ touched by $C$.
    Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.
4. Recurse on $G_1, \ldots, G_k$ starting from $v_i$
5. Splice together.
   1,10,7,8,5,10 ,8,4,3,11,4 5,2,6,9,2 and to 1!

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# A few details.

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Whatever you think best for you.

# TA/Professor Ramchandran opinion

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